11-4 Inequalities

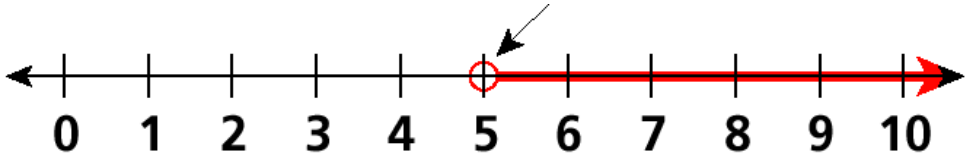
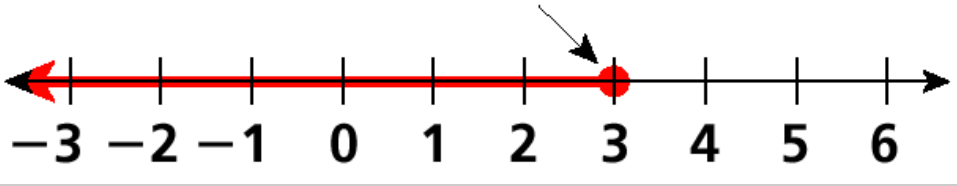
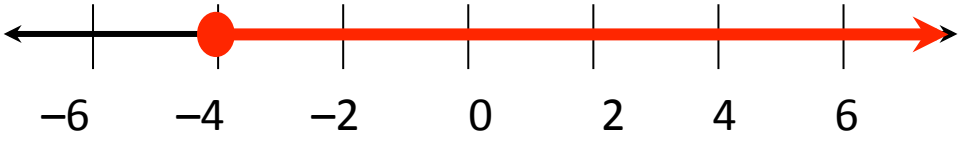
An **inequality** states that two quantities either are not equal or may not be equal. An inequality uses one of the following symbols:

Symbol	Meaning	Word Phrases	Graph	Example
$<$	**Less** than	**Fewer** than, below		
$>$	**Greater** than	**More** than, above		
$\leq$	Less than or **equal** to	At **most**, no more than		
$\geq$	Greater than or **equal** to	At **least**, no less than		

11-4 Inequalities

- An inequality that contains a variable is an **algebraic inequality**.
- An inequality may have more than one solution. Together, all of the solutions are called the **solution set.**
- You can graph the solutions of an inequality on a **number line.**
- If the variable is **"greater than"** or **"less than"** a number, then that number is indicated with an **open** circle.
- If the variable is **"greater than or equal to"** or **"less than or equal to"** a number, then that number is indicated with a **closed** circle.

11-4 Inequalities

Symbol	Graph	Example
< (open circle)		n < 3
> (open circle)		a > 5
≤ (closed circle)		b ≤ 3
≥ (closed circle)		a ≥ –4

11-5 Solving Inequalities by Adding or Subtracting

When you **add** or **subtract** the same number on both sides of an inequality, the resulting statement will still be true.

You can find solution sets of **inequalities** the same way you find solutions of **equations**, by isolating the **variable**.

Solve. Then graph the solution set on a number line.

A. $n - 7 \leq 15$

$$\begin{array}{rcl} n - 7 & \leq & 15 \\ +7 & & +7 \\ \hline n & \leq & 22 \end{array}$$

Add 7 to both sides.

Draw a closed circle at 22 then shade the line to the left of 22.

Solve. Then graph the solution set on a number line.

B. $a - 10 \geq -3$

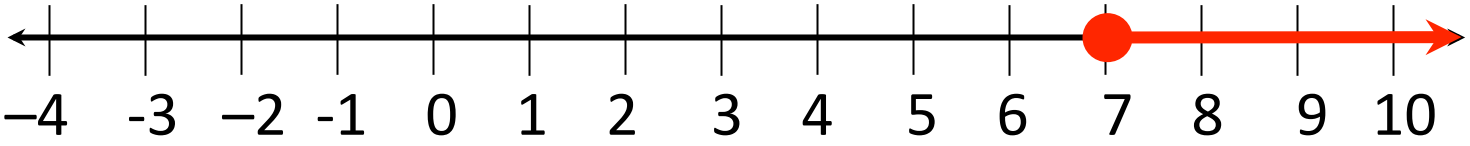
$a - 10 \geq -3$

$+10 \quad +10$

Add 10 to both sides.

$a \geq 7$

Draw a closed circle at 7.
Then shade the line to the right.



You can see if the **solution** to an inequality is true by choosing any number in the solution set and **substituting** it into the **original** inequality.

Check: *Solution:* $n \leq 22$

Substitute in a number less than or equal to 22.

A. $n - 7 \leq 15$

$10 - 7 \leq 15$

We choose 10.

$3 \leq 22$ ✓

Check: *Solution:* $a \geq 7$

Substitute in a number greater than or equal to 7

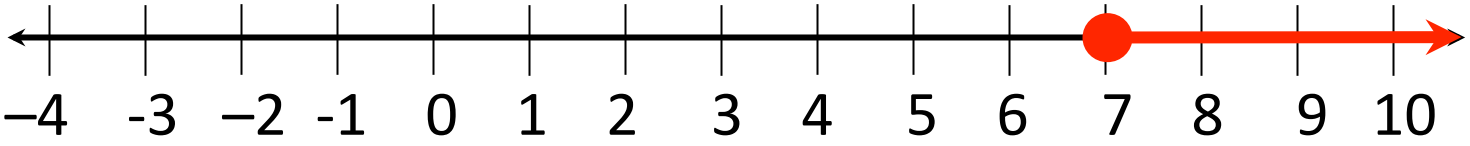
B. $A + -10 \geq -3$

We choose 12.

$12 + -10 \geq -3$

$2 \geq -3$

$2 \geq -3$ ✓



11-6 Solving Inequalities by Multiplying or Dividing

When you **multiply** or **divide** both sides of an inequality by the same **positive** number, the statement will still be **true**.

When you **multiply** or **divide** both sides by the same ***negative*** number, you need to **reverse** the direction of the inequality symbol for the statement to be true.

$$-4 < 2$$

$$(3)(-4) < (3)(2)$$

$$-12 < 6$$

$$-4 < 2$$

$$(-3)(-4) > (-3)(2)$$

$$12 > -6$$

Solve and check.

C. $\frac{c}{4} \leq -4$

$\frac{c}{4} \leq -4$

$(4)\ \frac{c}{4} \leq (-4)\ (4)$

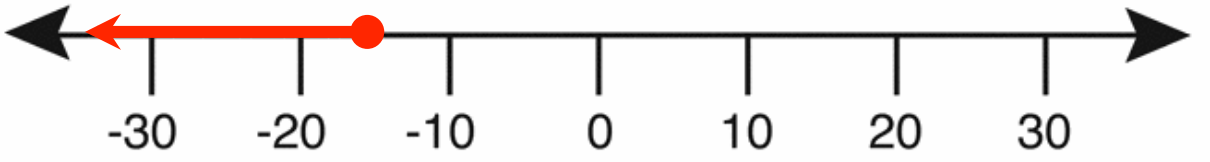
$c \leq -16$

Check:

$\frac{c}{4} \leq -4$

$\frac{-20}{4} \leq -4$

$-5 \leq -4$ ✓



Solve. Check your answer.

D. $-7b < 56$

$$\frac{-7b}{-7} > \frac{56}{-7}$$

Divide both sides by –7, and reverse the inequality symbol.

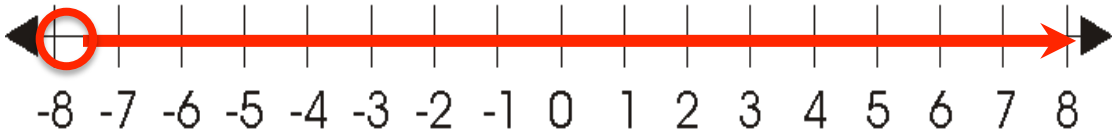
$$b > -8$$

Check:

$-7b < 56$

$-7(0) < 85$

$0 < 85$ ✓



11-7 Solving Two-Step Inequalities

When you solve two-step and multi-step **equations**, you can use the order of operations in **reverse** to isolate the **variable**.

You can use the same process when solving two-step and multi-step **inequalities**.

Solve. Then graph the solution set on a number line.

E. $\frac{y}{2} - 6 > 1$

$\frac{y}{2} - 6 > 1$

$+6 \quad +6$

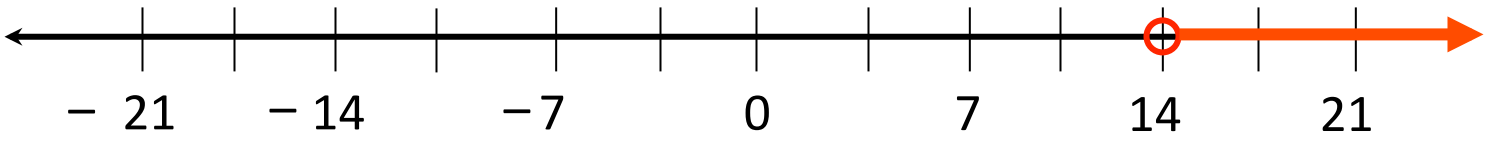
$\frac{y}{2} > 7$

Add 6 to both sides.

$(2)\ \frac{y}{2} > 7\ (2)$

Multiply both sides by 2.

$y > 14$



Check: $y > 14$

E. $\frac{y}{2} - 6 > 1$

$\frac{20}{2} - 6 > 1$

$10 - 6 > 1$

$4 > 1$ ✓

Choose any number greater than 14 to substitute in and check!

Solve. Then graph the solution set on a number line.

F. $-9x + 4 \leq 31$

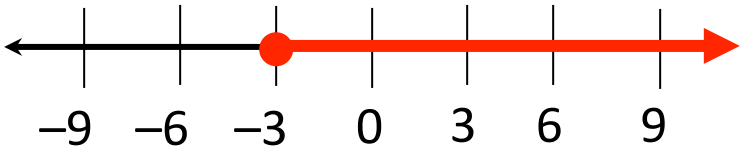
$-9x + 4 \leq 31$

$-4 \quad -4$ *Subtract 4 from both sides.*

$-9x \leq 27$

$\frac{-9x}{-9} \geq \frac{27}{-9}$ *Divide both sides by –9, and reverse the inequality symbol.*

$x \geq -3$



Check: $x \geq -3$

F. $-9x + 4 \leq 31$

$-9\,(0) + 4 \leq 31$

$0 + 4 \leq 31$

$4 \leq 31$ ✓

Choose a number greater than or equal to -3 to substitute in for x.

11-7 Solving Two-Step Inequalities

Solve. Then graph the solution set on a number line.

G. $3(w + 7) < -5w - 3$

$3(w + 7) < -5w - 3$

$3w + 21 < -5w - 3$

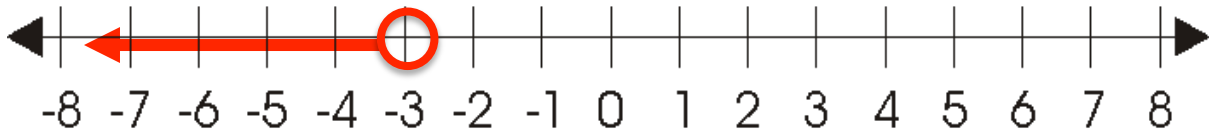
$+5w \qquad +5w$

$8w + 21 < -3$

$-21 \qquad -21$

$\frac{8w}{8} < \frac{-24}{8}$

$w < -3$



Check: $w < -3$

G. $3(w + 7) < -5w - 3$

$3(-5 + 7) < -5(-5) - 3$

$3(2) < -5(-5) - 3$

$6 < -5(-5) - 3$

$6 < 25 - 3$

$6 < 22$ ✓

Choose a number less than -3 to substitute in for w.